Mathematics Mastery

5-week maths pack

Guidance on using these resources:

We have mapped out five weeks of tasks consisting of four sessions in each week.

Each session is designed to last 1 hour and consists of:

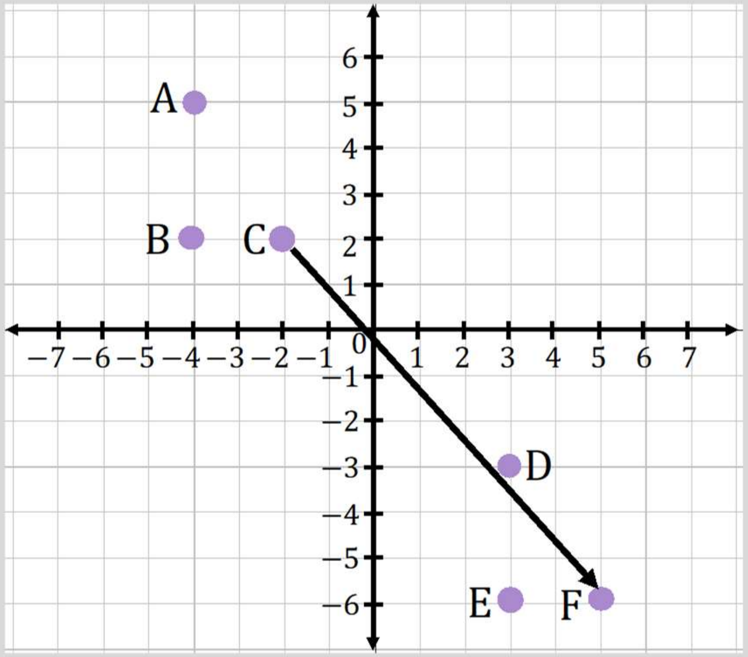
- Two tasks contained within this pack (20 minutes)
- A practice exercise of questions linked to the tasks (40 minutes)

The focus for each of the weeks and the session titles are shown in the timetable below:

	Session title		**Session title**
Week 1: Transformations	Translation	Week 4: Prime factorisation 2	Highest common factor
	Rotation		More highest common factor
	Reflection		Lowest common multiple
	Isometries		More lowest common multiple
Week 2: More transformations	Combining reflections	Week 5: Fractions	Part of a whole
	Combining translations and reflections		Fractions of measure
	Enlargements		Fair shares
	Enlargement and area		Equivalence
Week 3: Prime factorisation 1	Indices		
	Prime factors		
	Prime factorisation		
	Using the prime factorisation		

Week 1 Session 1: Translation

Translations are movements in a direction.
Column vectors can be used to describe translations.



7 units in the positive x-direction

C to F : $\begin{pmatrix} 7 \\ -8 \end{pmatrix}$

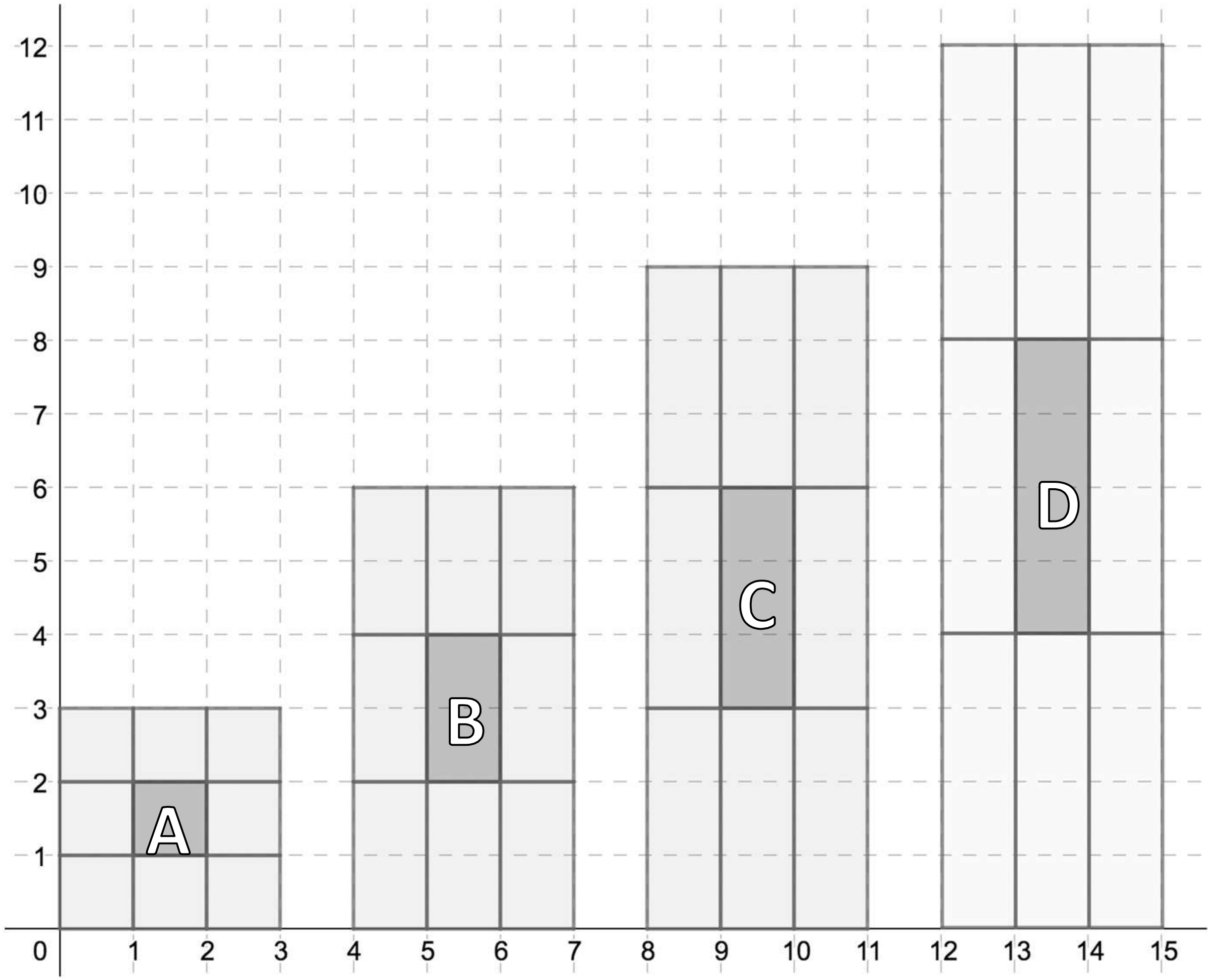
8 units in the negative y-direction

Write a vector that can describe the translations :

F to C D to B B to C A to B

Week 1 Session 1: Translation

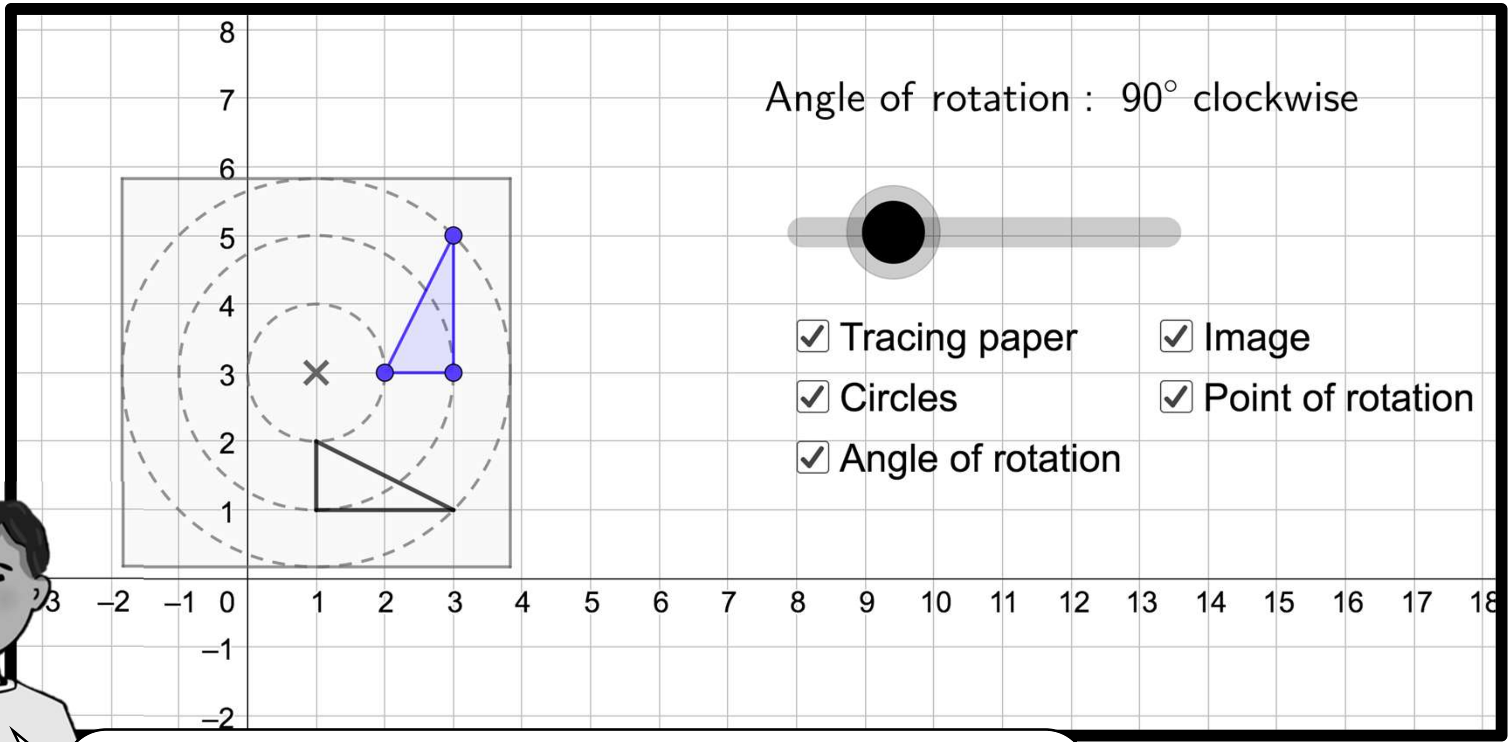
Describe the translations from the central rectangle to the surrounding rectangles in each case.



How could you continue this pattern?

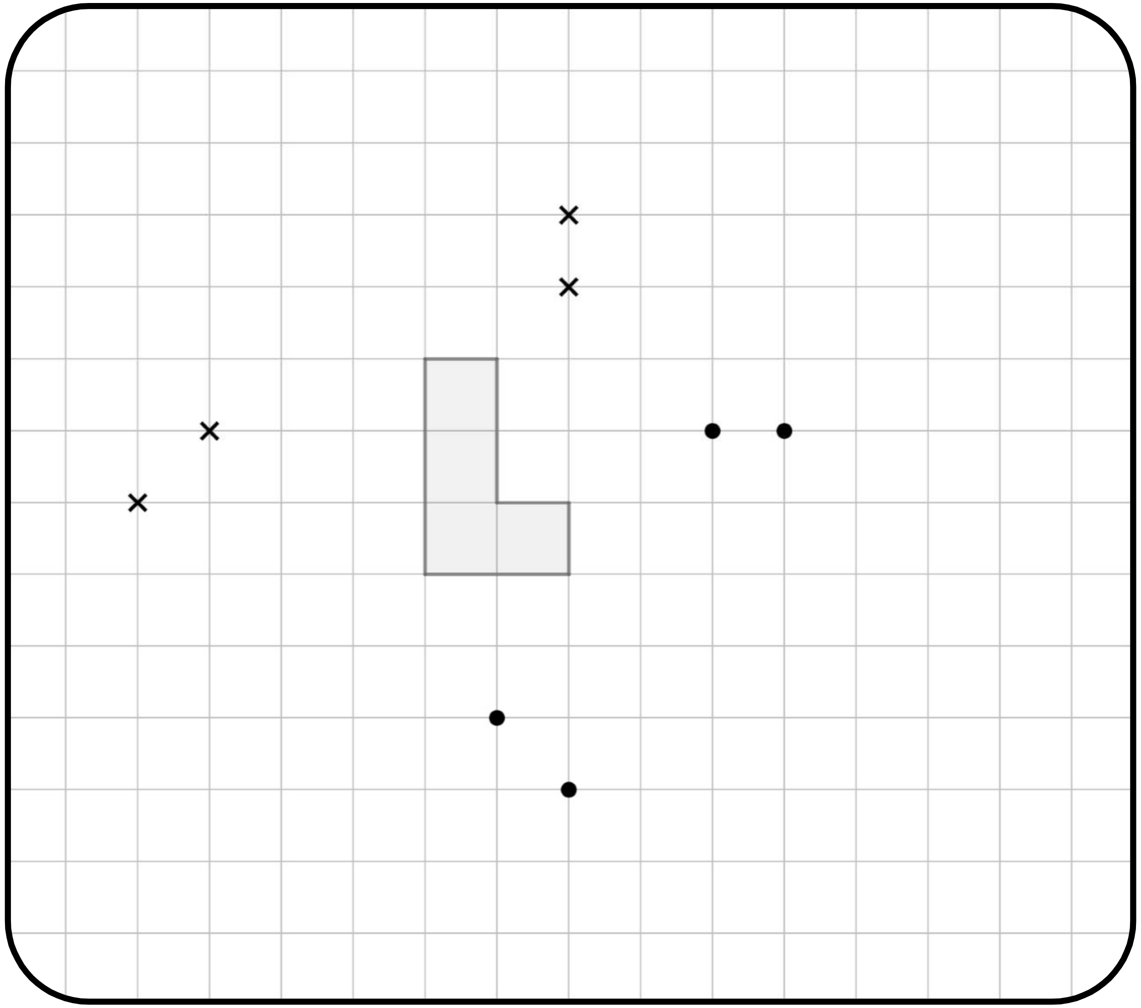
Week 1 Session 2: Rotation

We can rotate shapes about a **point of rotation**.



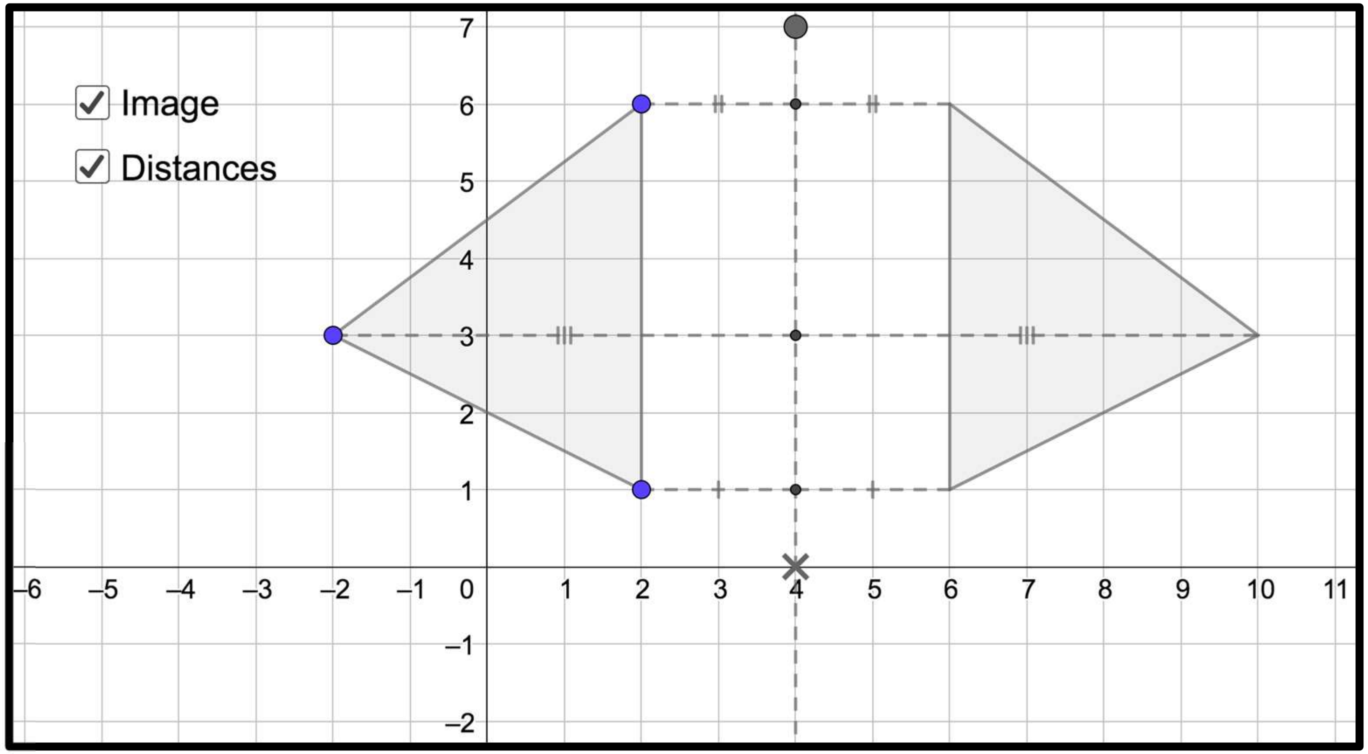
Access the Geogebra file on this link:
https://www.geogebra.org/classic/vneqtrtf

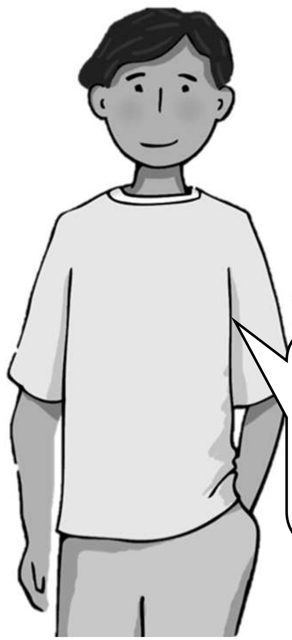
Copy this image, rotate the hexagon $90°$ clockwise about the black crosses and $90°$ anticlockwise about the black dots.



I wonder how moving the point of rotation will affect the image.

We can reflect shapes in a line of reflection.
Points and their reflections will be equidistance from this line.



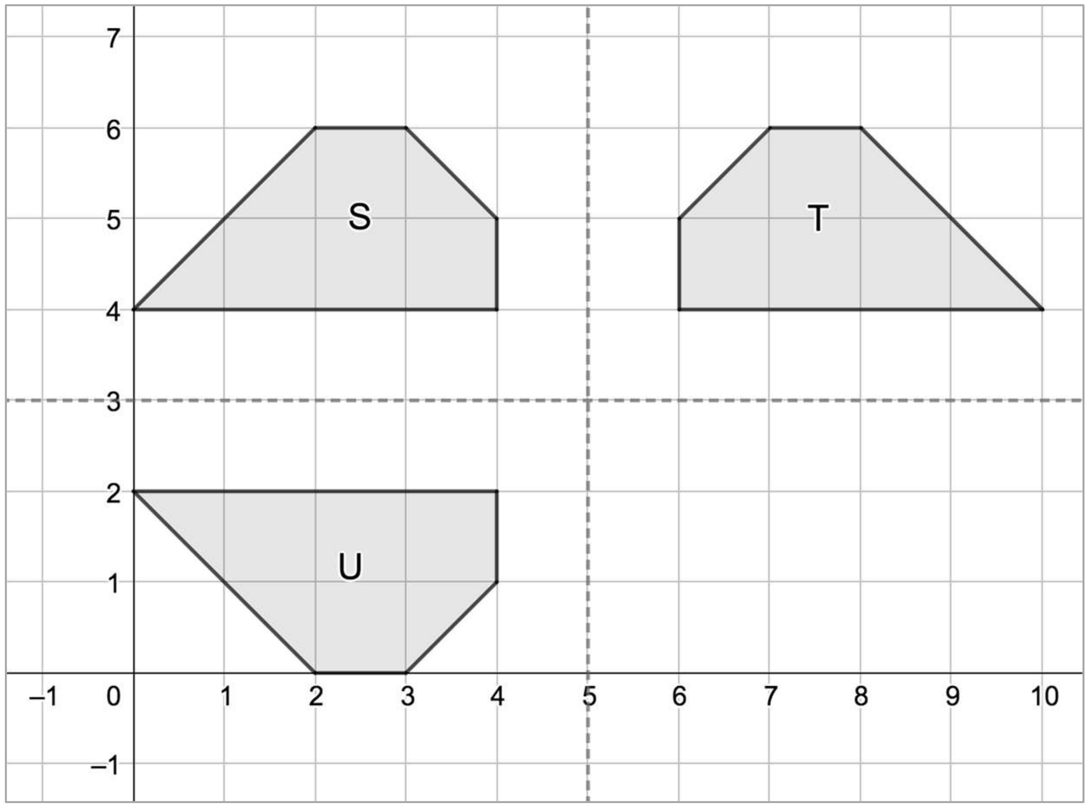


Access the Geogebra file on this link:
https://www.geogebra.org/classic/c5fmvhfw

T and U are reflections of S. What are the lines of reflection?

Explore the effect on the **reflected images** if **S is translated** by the vectors:

$\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}0\\1\end{pmatrix}$ $\begin{pmatrix}1\\-1\end{pmatrix}$



Week 1 Session 4: Isometries

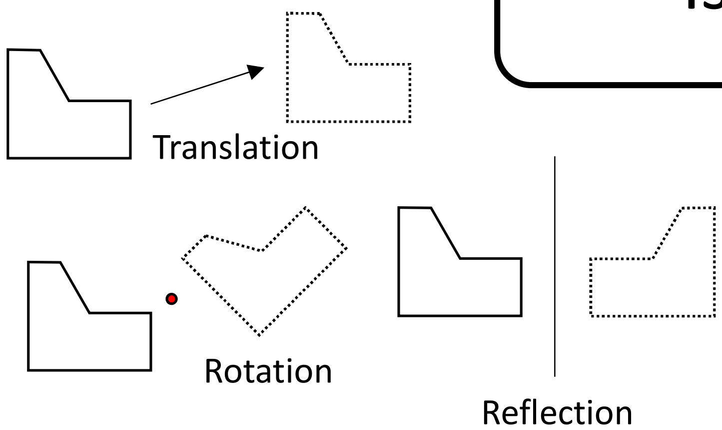
Isometry

Definition

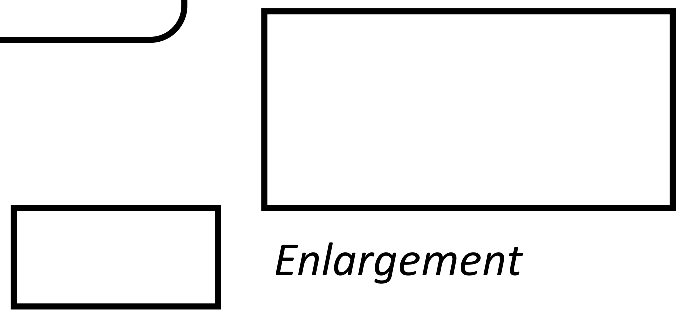
*Isometries are **transformations** that do not affect the size or shape of an object.*

Characteristics

The object and the image are the same shape (congruent). They can be turned over, turned round or moved without turning or flipping.



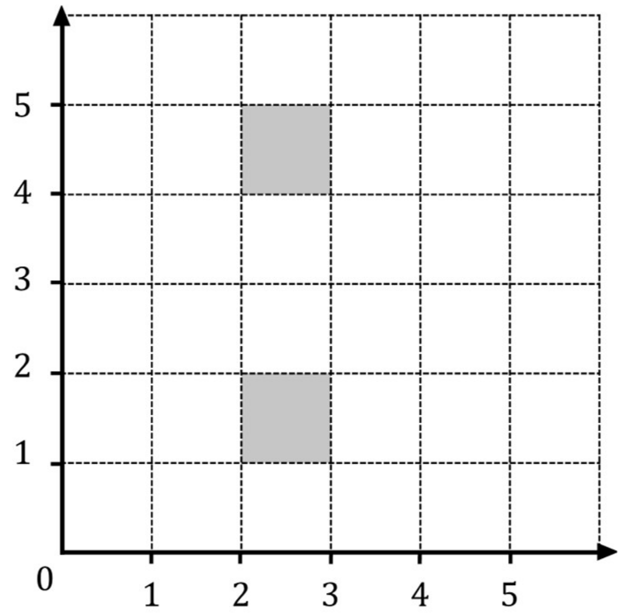
Examples



Non-examples

Week 1 Session 4: Isometries

There are different ways to transform one of the squares onto the other. Complete the descriptions:



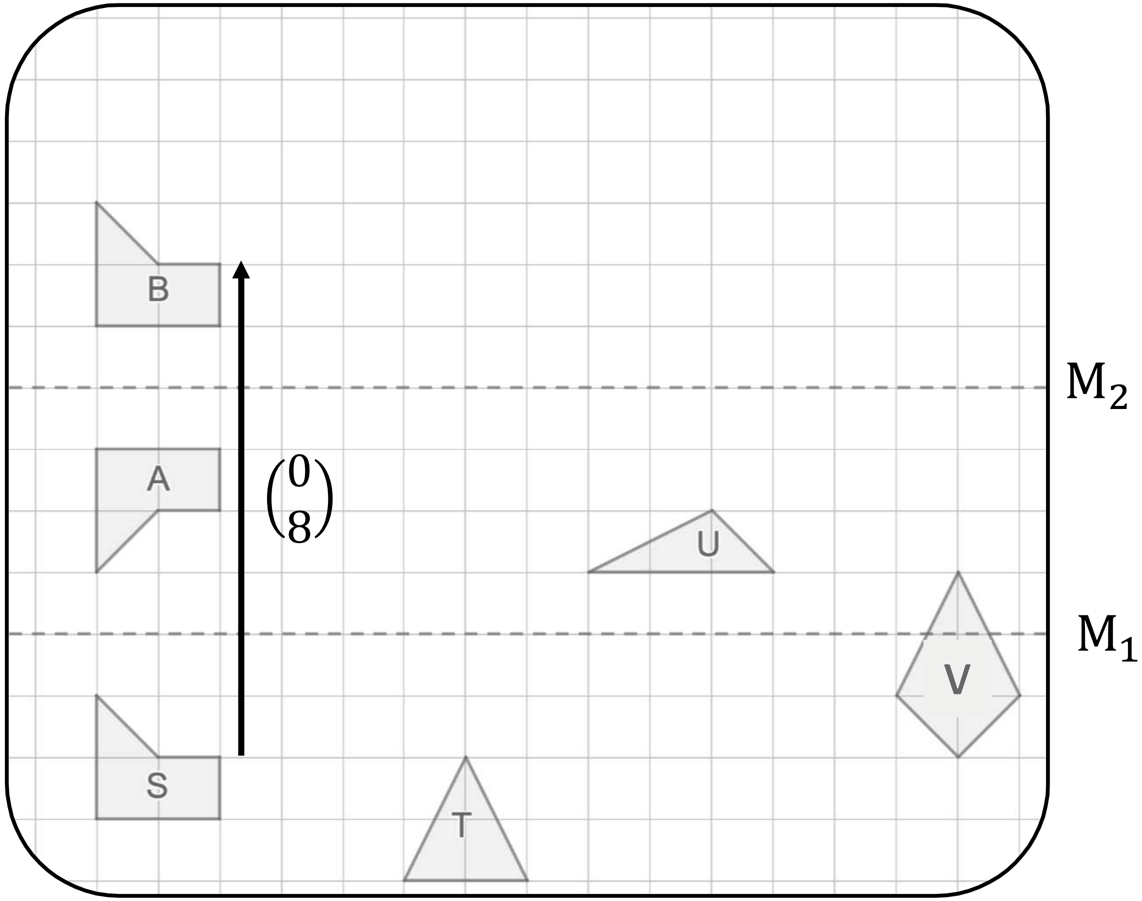
I reflected …

I rotated …

I translated …

We can sometimes describe the effect of **combining** transformations using a single transformation.

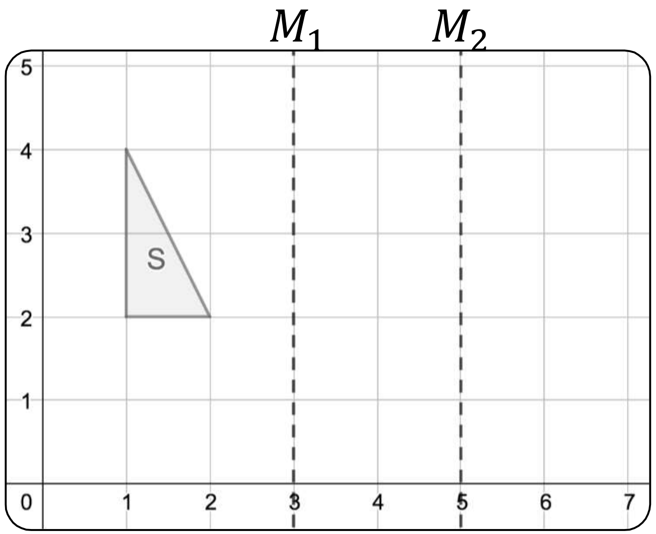
Reflecting S in M_1 then in M_2 has the same effect as a translation

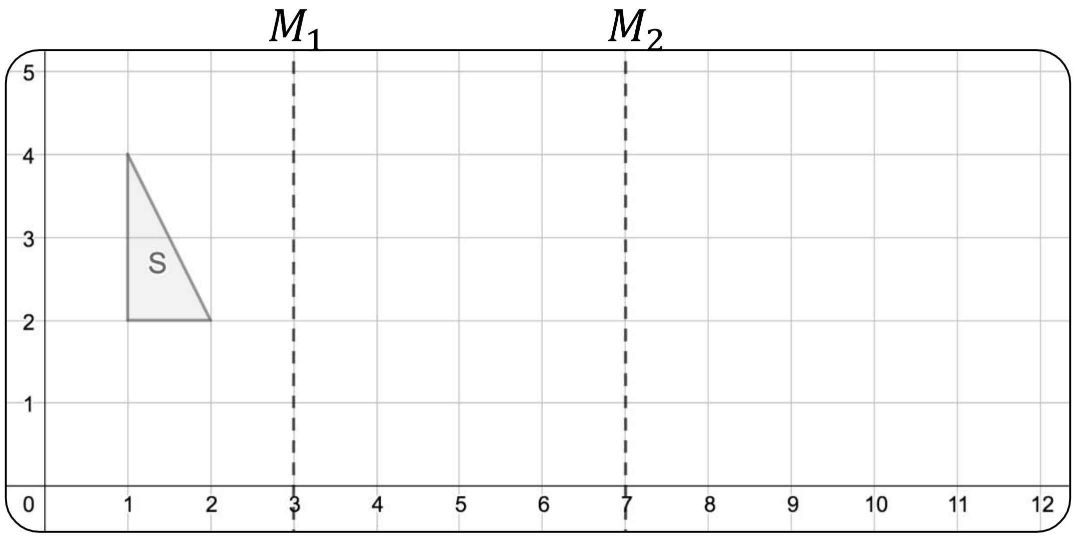


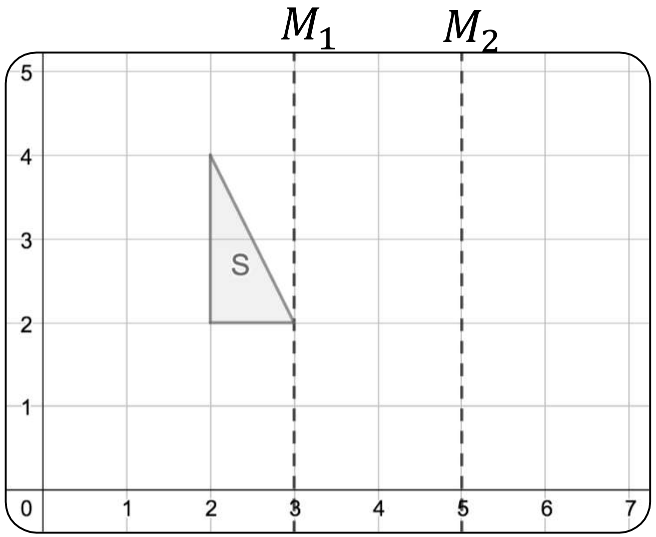
Explore the effect of this combination of reflections for: T, U and V.

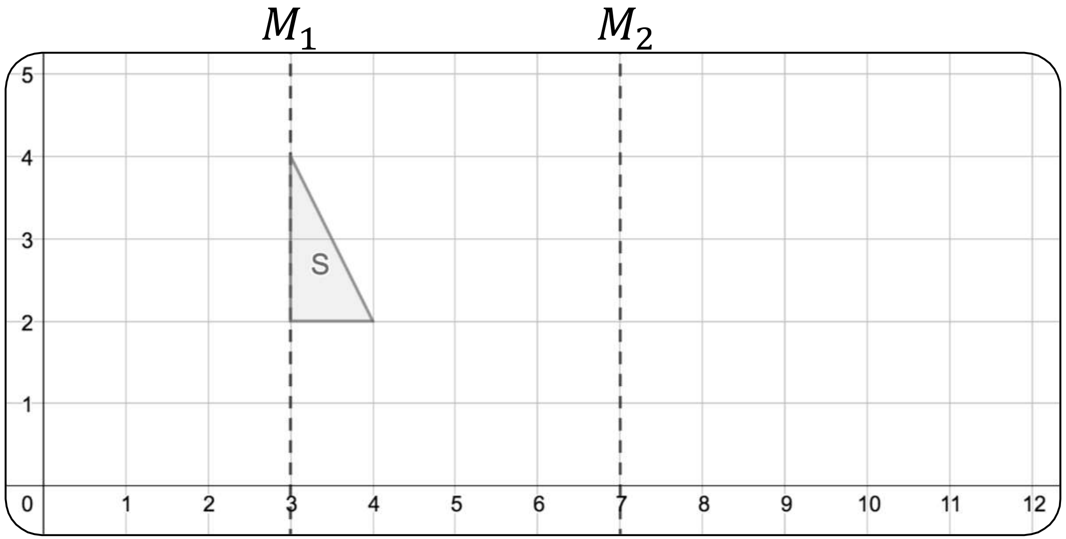
Week 2 Session 1: Combining reflections

Reflect S in M_1 then in M_2 . Describe the single transformation from S to to the final image.



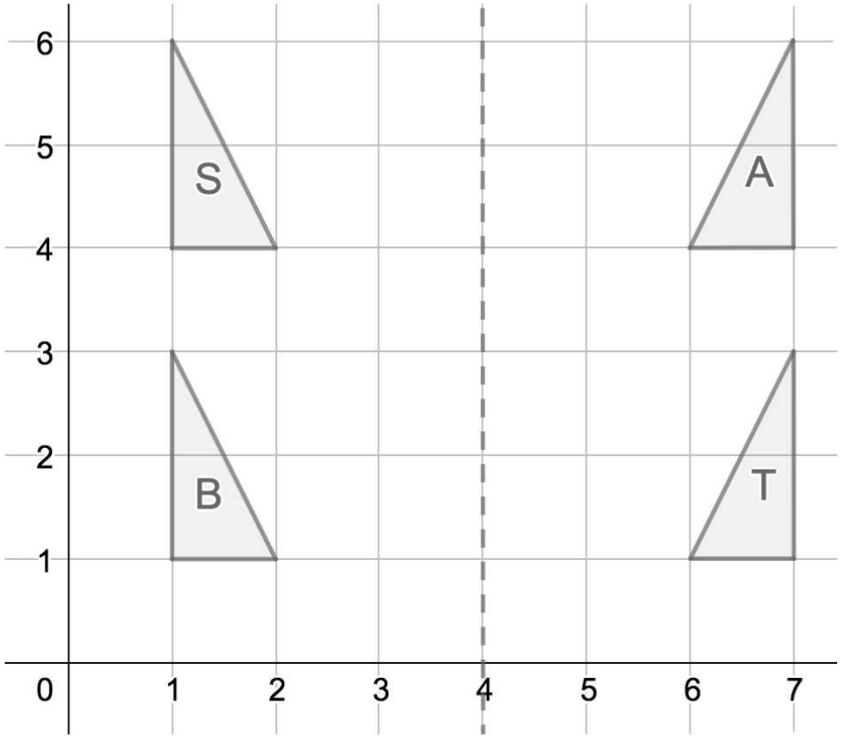






Week 2 Session 2: Combining translations and reflections

Describe the transformation, or combination of transformations, between each pair of triangles:

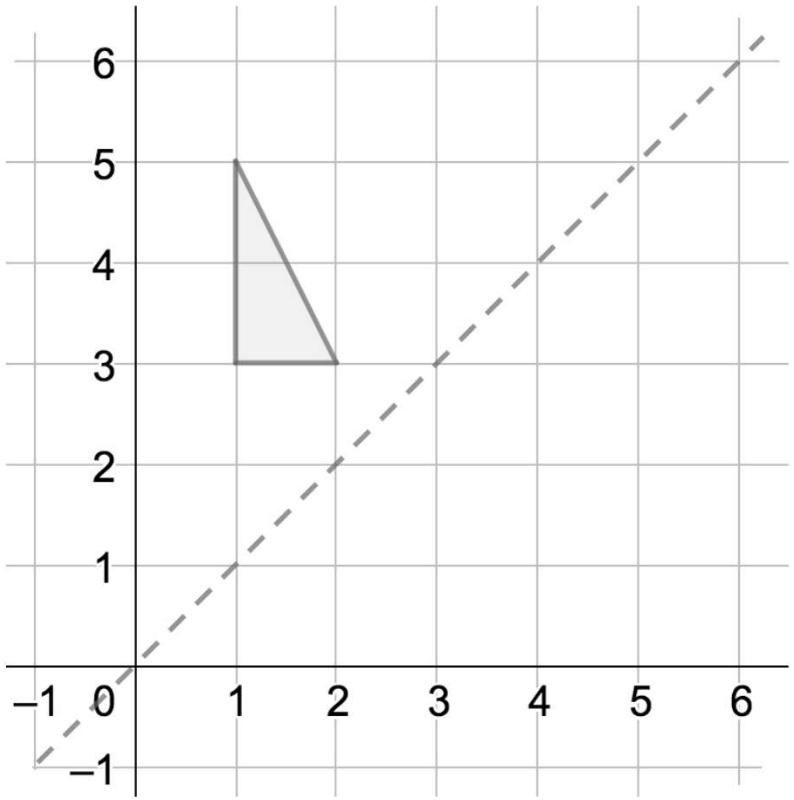


e.g. Triangle T is the reflection of triangle S in the line $x = 4$ followed by a translation by the vector $\binom{0}{-3}$

Week 2 Session 2: Combining translations and reflections

Using the line of symmetry shown, compare the effect of...

- reflecting then translating
- translating then reflecting



... for each of the vectors:

$\begin{pmatrix}1\\0\end{pmatrix}$

$\begin{pmatrix}0\\-3\end{pmatrix}$ $\begin{pmatrix}1\\2\end{pmatrix}$ $\begin{pmatrix}-2\\-2\end{pmatrix}$

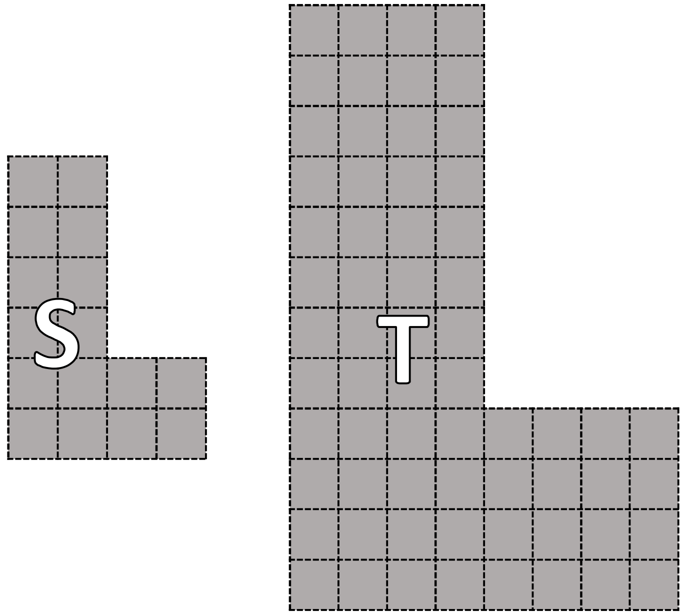
$\begin{pmatrix}1\\1\end{pmatrix}$

Week 2 Session 3: Enlargement

Enlargements of shapes can be described using **scale factors.**

T is an enlargement of S by a scale factor 2

S is an enlargement of T by a scale factor $\frac{1}{2}$

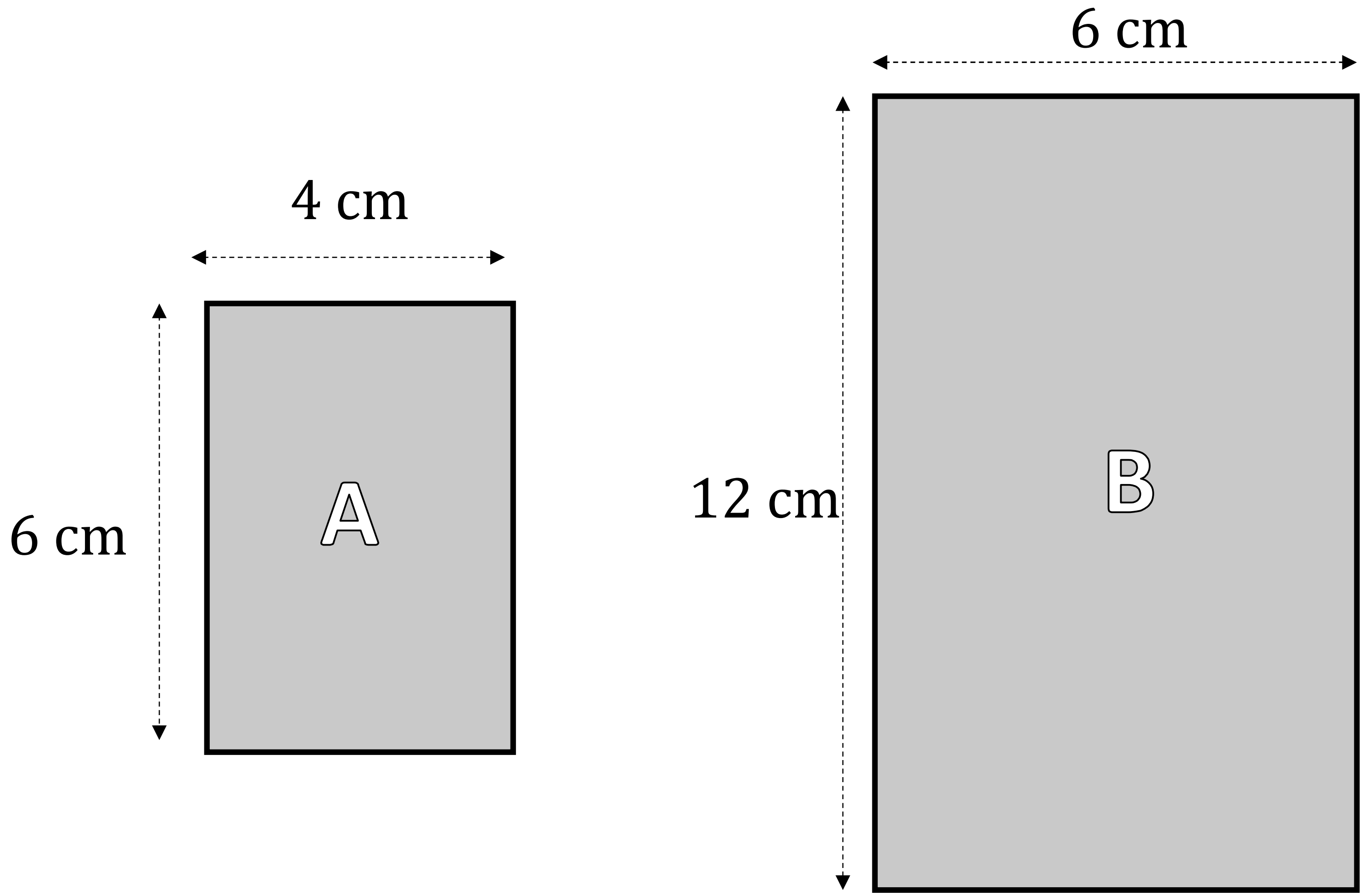


Draw S following an enlargement of scale factor:

3 1 $\frac{1}{2}$ 0

Week 2 Session 3: Enlargement

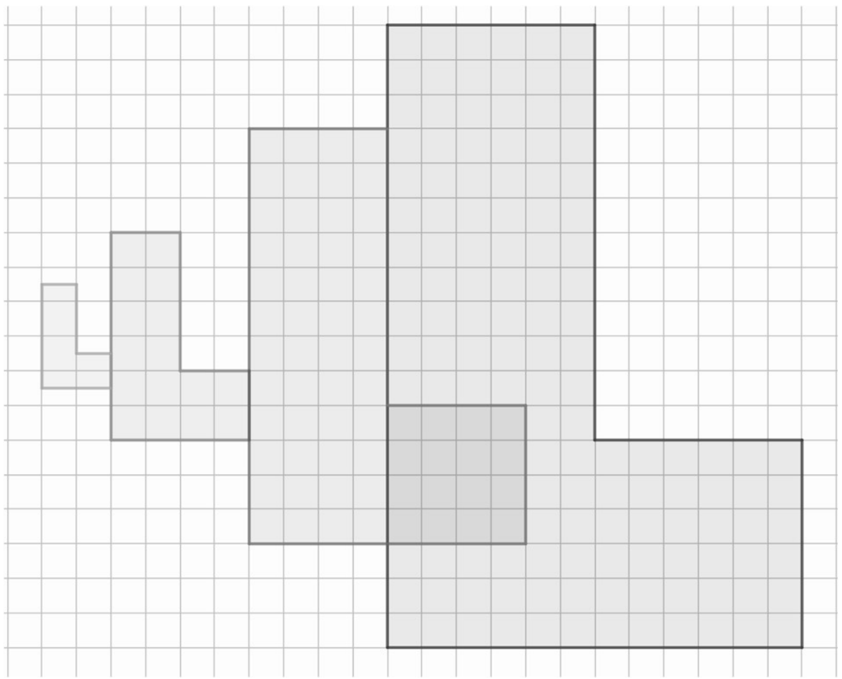
Explain why B is not an enlargement of A.



How could you change **one of the dimensions** of A or B so that it is?

Week 2 Session 4: Enlargement and area

When a shape is enlarged the **area** is affected.



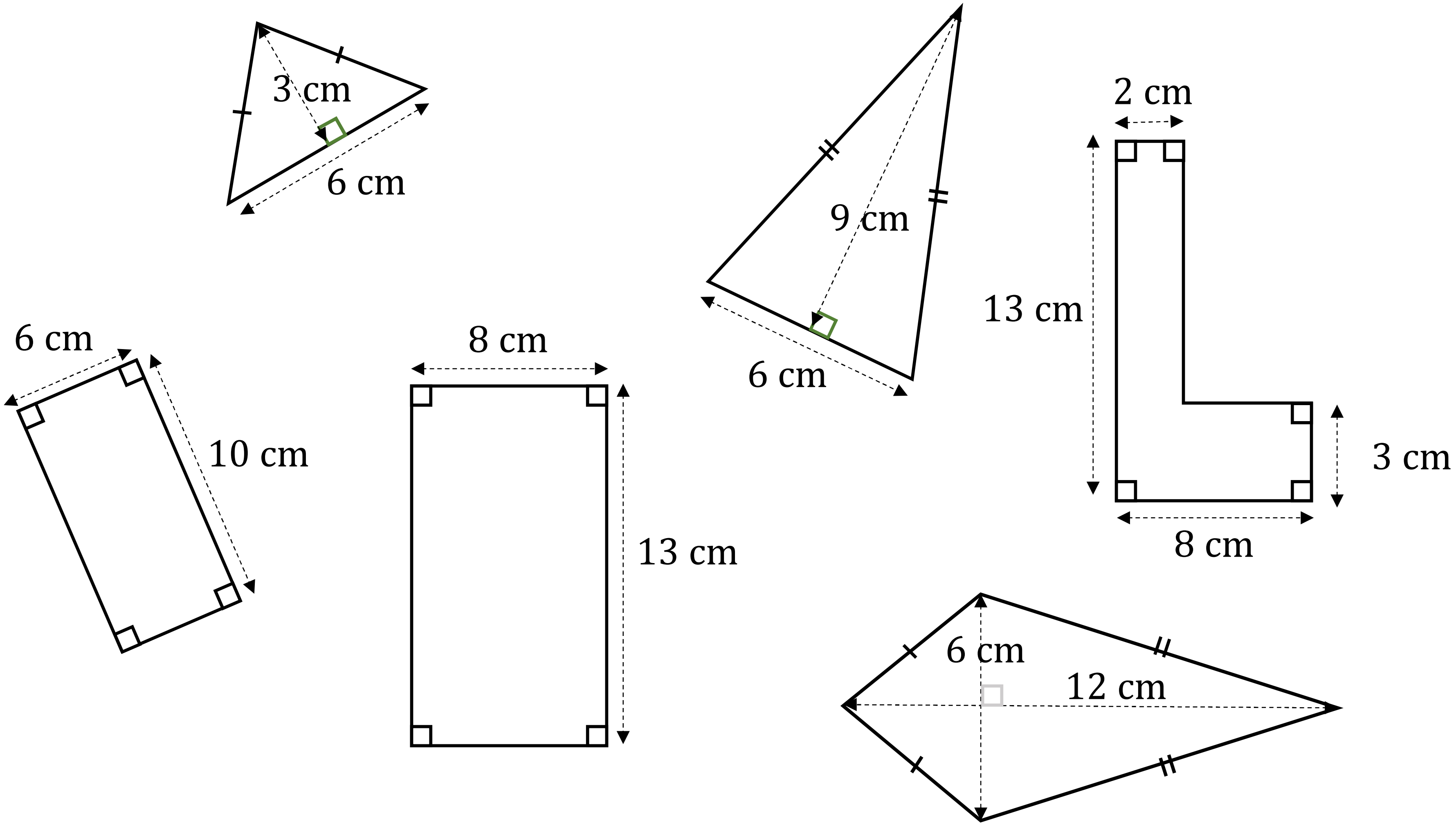
Find the scale factor of enlargement between the different shapes.

How has the area been affected in each case?

Week 2 Session 4: Enlargement and area

Draw **sketches** of the following shapes after they’re enlarged by a scale factor 7.

How do enlargements affect the areas?



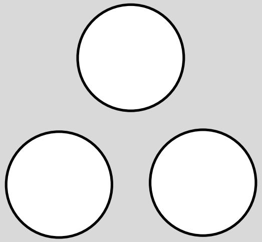
Week 3 Session 1: Indices

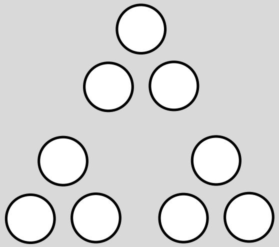
We can use index notation to describe repeated products.

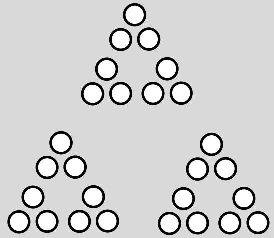
$3^1 = 3$

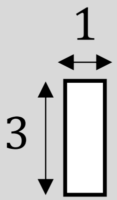
$3^2 = 3 \times 3$

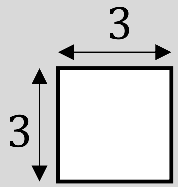
$3^3 = 3 \times 3 \times 3$

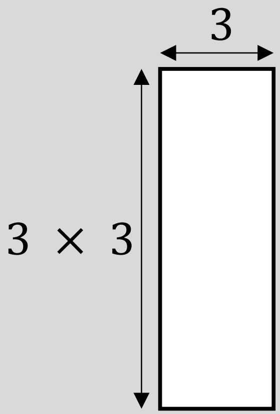










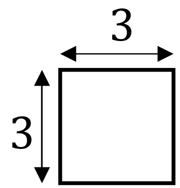


Connect each representation to the calculation.

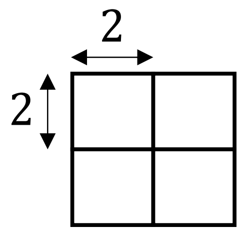
How could you represent 3^4?

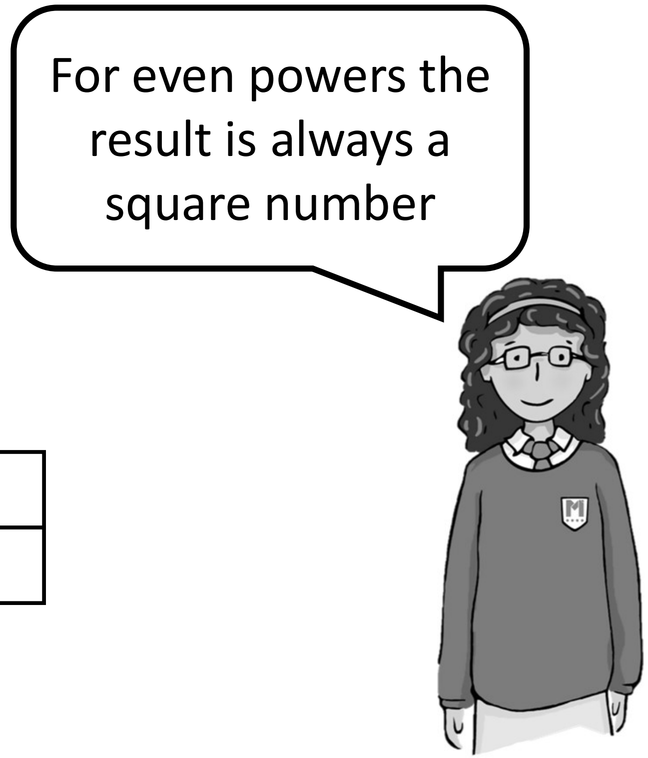
Is this student correct? Test out her conjecture by trying out other powers.
TIP: You should use at least 8 calculations.

$$3^2 = 3 \times 3 = 9$$



$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$





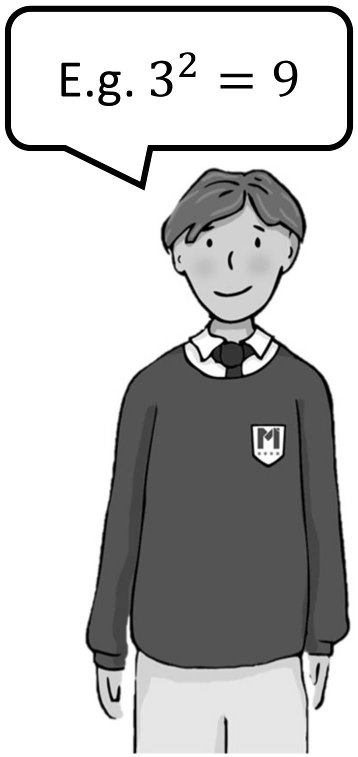
Test out some conjectures of your own.
For example, *odd numbers raised to any power are always odd*
OR *the final digit of a power of 2 is always a 2,4,6 or 8*.

Week 3 Session 2: Prime factors

If you multiply ONLY 1s and 2s, you can make the products 1,2,4,8,16,32 and 64. These have been shaded grey on the grid.

Write down (or circle on the grid) the other numbers that could you make if you are now able to multiply together combinations of 1s, 2s and 3s.
TIP: you do not have to use 1,2 and 3 in each calculation.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



These students are discussing what happens when you include 4s:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

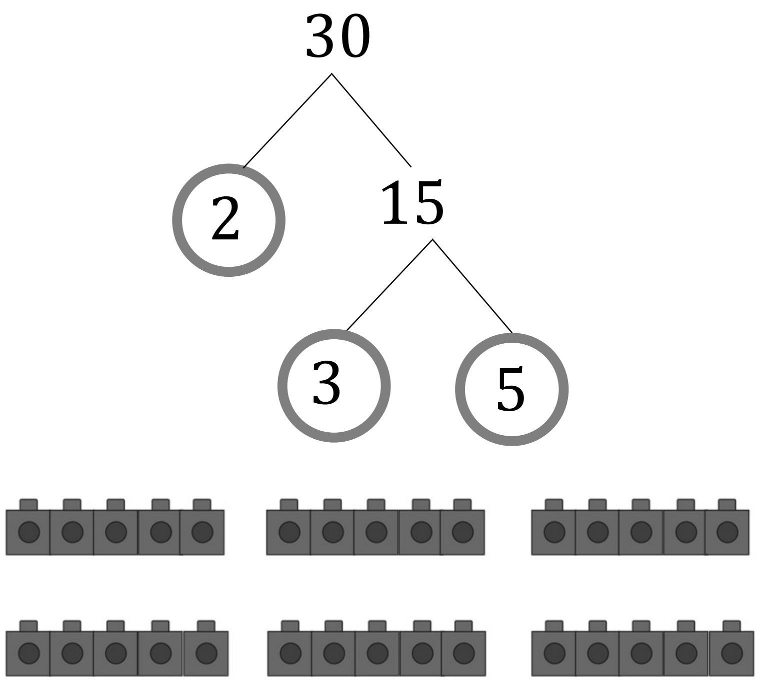
Do you agree? Explain your answer.

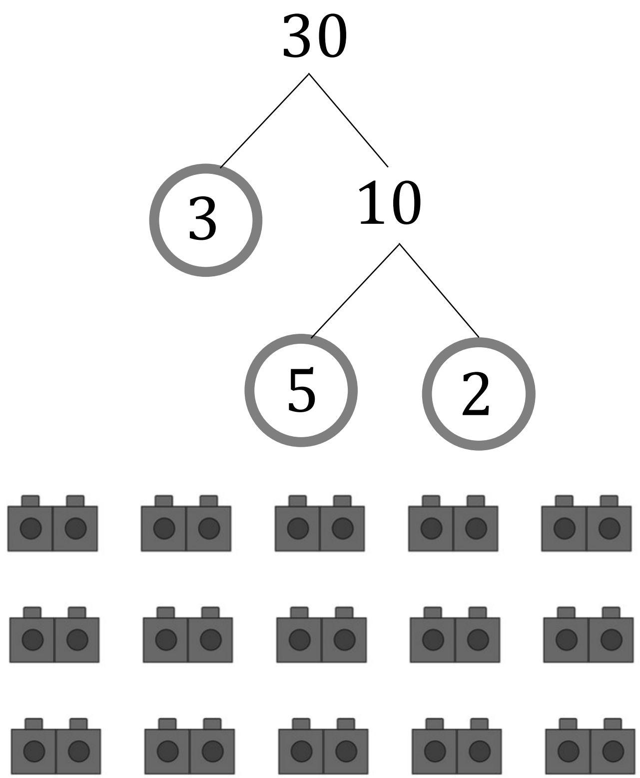
Explain what will change when 5s are included.

Week 3 Session 3: Prime factorisation

Every compound integer can be written as a **product** of prime numbers.

$$\boxed{30 = 2 \times 3 \times 5}$$





What’s the same , what is different?

Week 3 Session 3: Prime factorisation

Write each of these as a product of prime numbers:

50	150	50×10	50^2
24	72	24×10	24^2

Week 3 Session 4: Using the prime factorisation

Here are all the factors of 60.
Write each one as a product of prime factors.

60	30	20	15	12	10
1	2	3	4	5	6

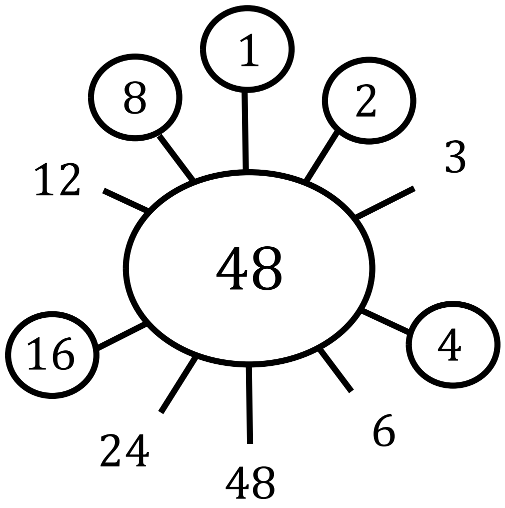
Compare the prime factorisation of 60 with the prime factorisation of its factors.
What do you notice?

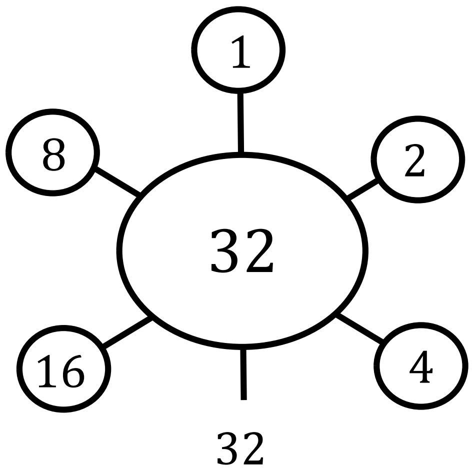
Help Phil to find **all** the factor pairs for this number:

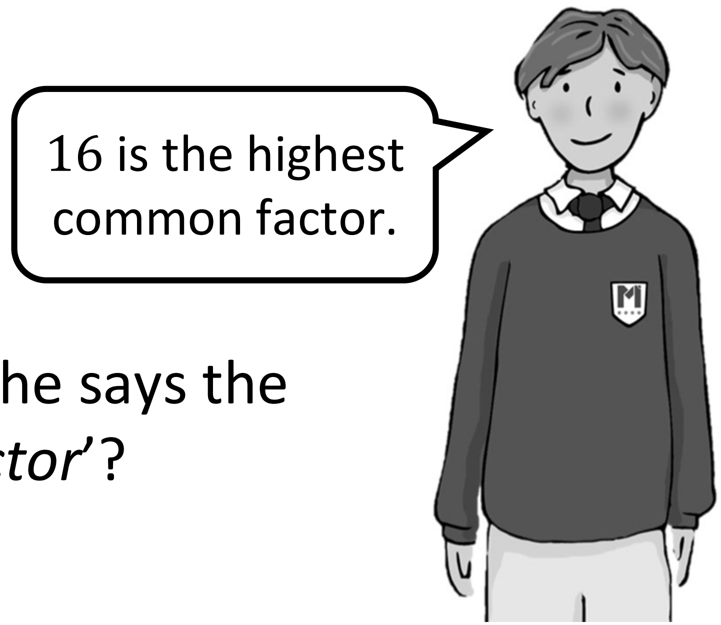
$$2 \times 5^2 \times 7$$

$2 \times 5 \times 7$	5
5×7	5×2
⋮	⋮

Two students are looking at the **common factors** of 48 and 32:





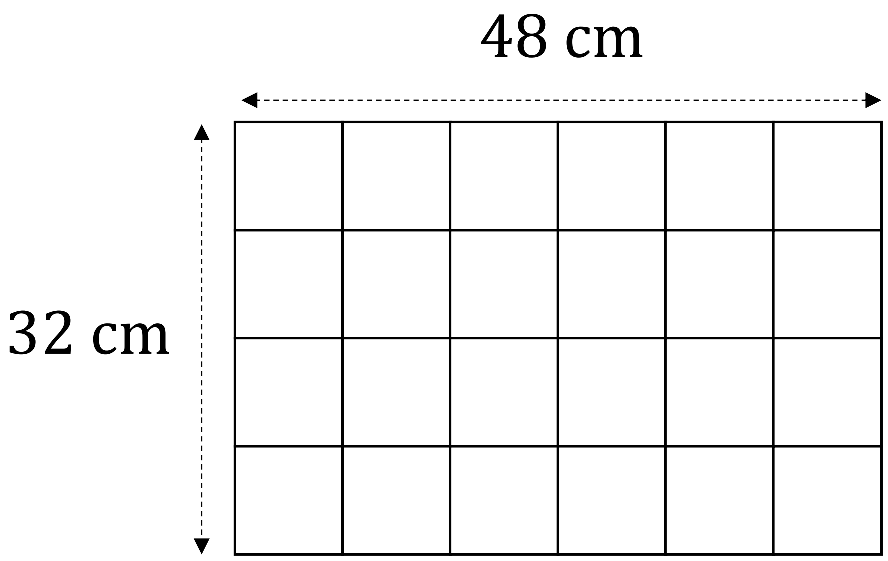


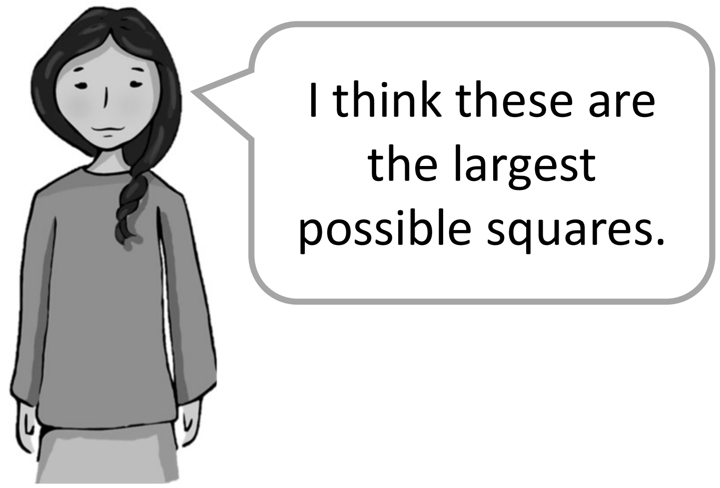
What does Ali mean when he says the *'highest common factor'*?

Week 4 Session 1: Highest common factor

This rectangle has been divided into identical squares.

What is the side length of the squares?





Do you agree with the student's statement?
What other sized squares can you divide it into?

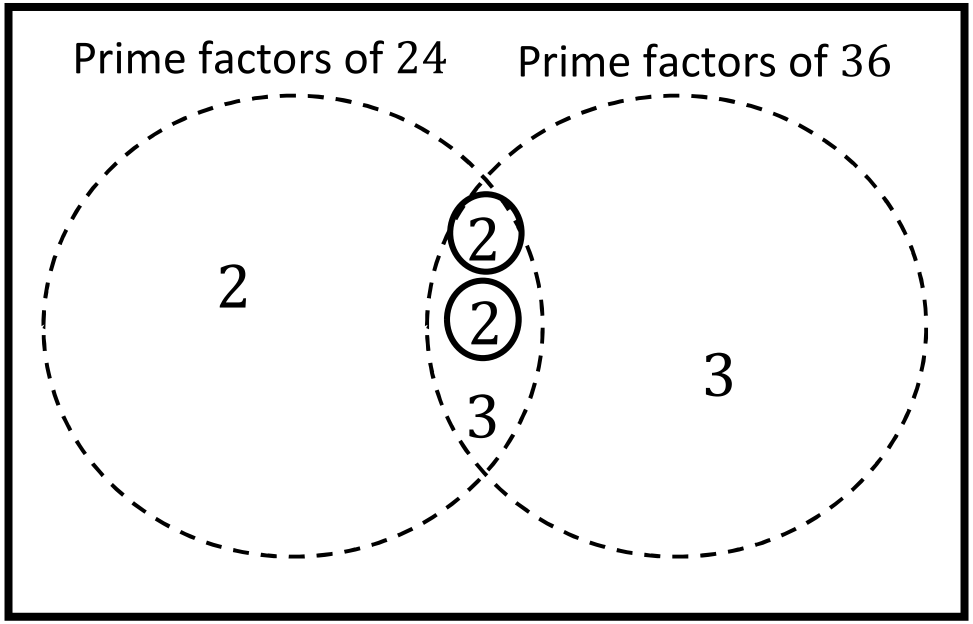
Similarly, explore the different sizes of identical squares that can fit in a 12 cm × 24 cm rectangle. What about an 18 cm × 24 cm rectangle?

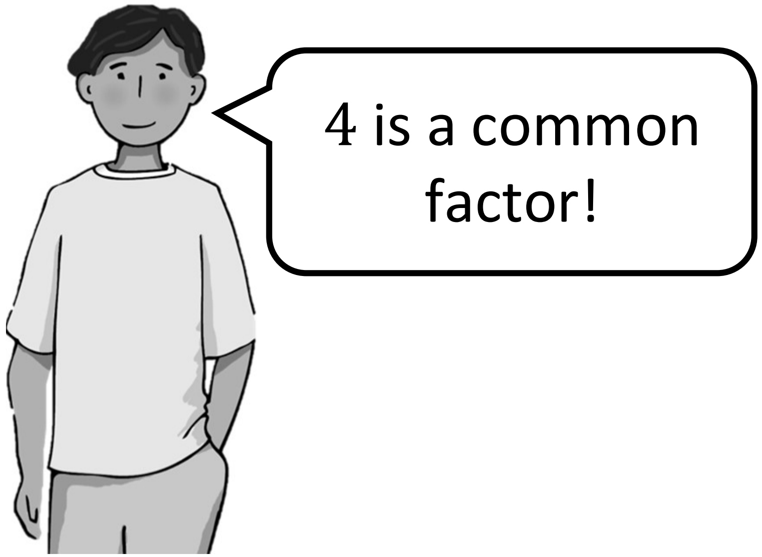
Week 4 Session 2: More highest common factor

Venn Diagrams can be used to identify common factors.
What common factors can you see?

$$24 = \underline{2 \times 2} \times 2 \times 3$$

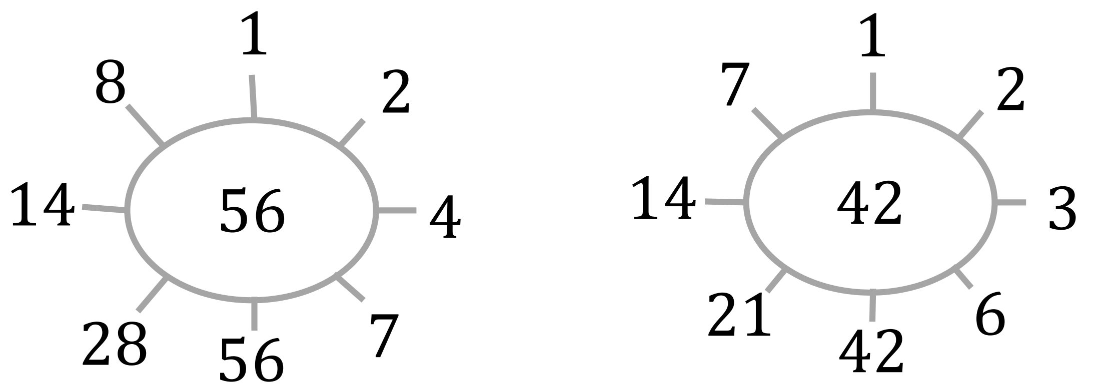
$$36 = \underline{2 \times 2} \times 3 \times 3$$

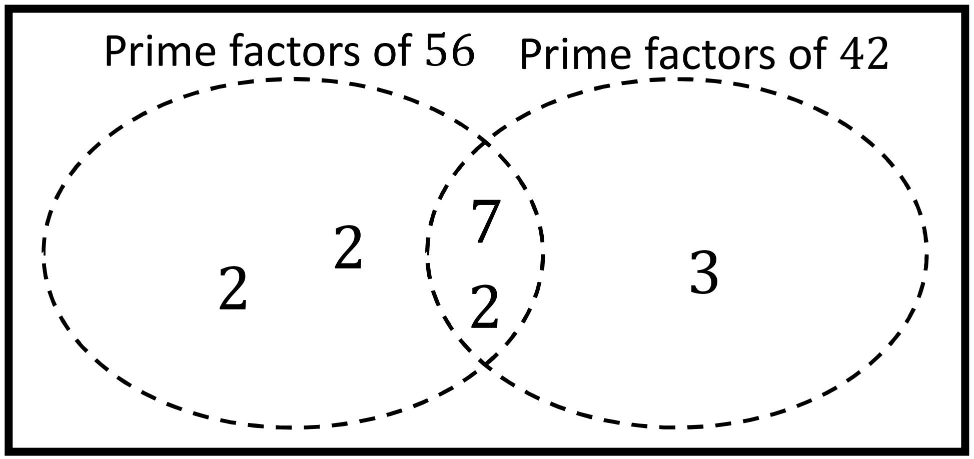




Week 4 Session 2: More highest common factor

Copy the diagrams below. Explain how each strategy can help find the HCF.



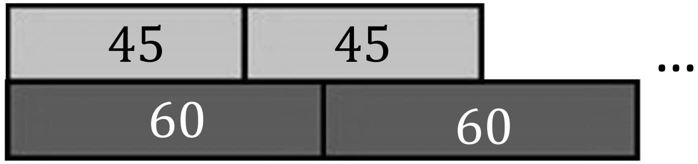


$$56 = 2 \times 2 \times 2 \times 7$$

$$42 = 2 \times 3 \times 7$$

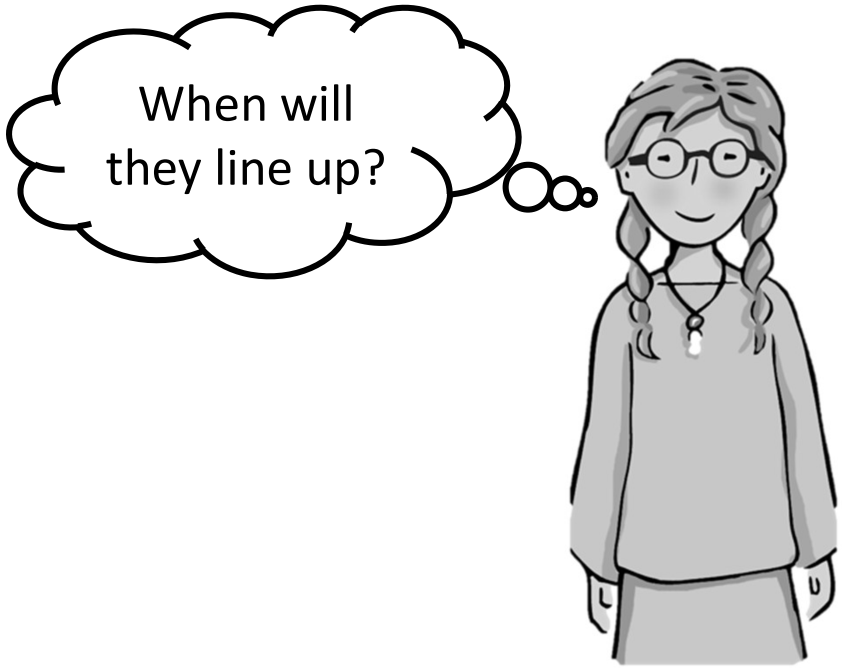
Week 4 Session 3: Lowest common multiple

Copy the diagram below. Continue the pattern to help Rosie find the **common multiples** of 45 and 60 up to 180.



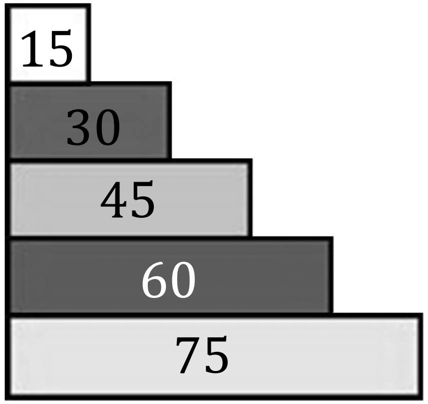
What is the lowest common multiple of 45 and 60 ?

Find examples for different numbers.



Week 4 Session 3: Lowest common multiple

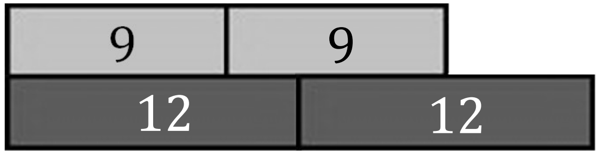
Select two of the numbers from the list below.
By drawing diagrams and/or listing numbers, find their lowest common multiple:



Repeat this for another **three** pairs of numbers.

What do you notice?

Week 4 Session 4: More lowest common factor



…

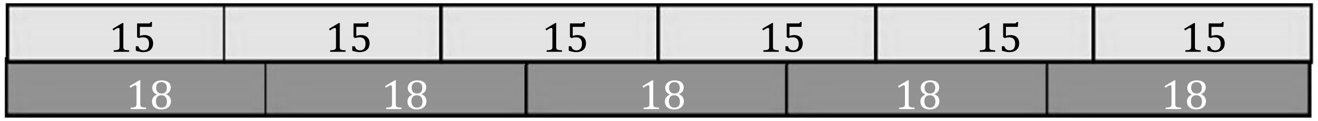
By continuing the pattern above, find the **first five** common multiples of 12 and 9.

Write each as a product of their prime factors.

What similarities are there between each of the product of prime factors?

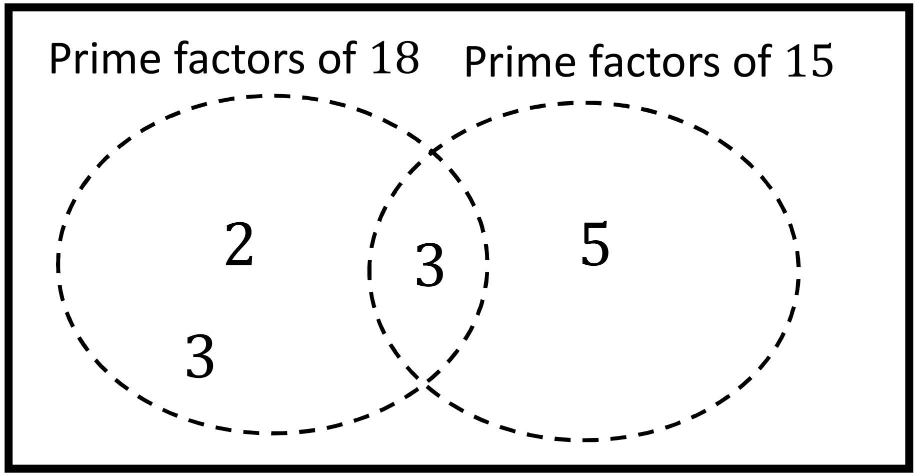
Week 4 Session 4: More lowest common multiple

Explain how each strategy can help find the LCM.



18 , 36 , 54 , 72 , 90 , 108 …

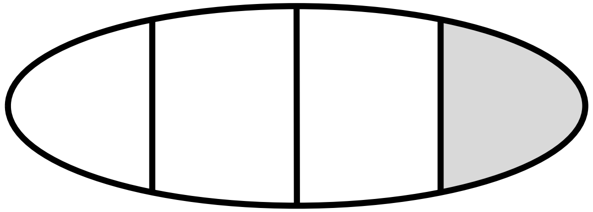
15 , 30 , 45 , 60 , 75 , 90 …



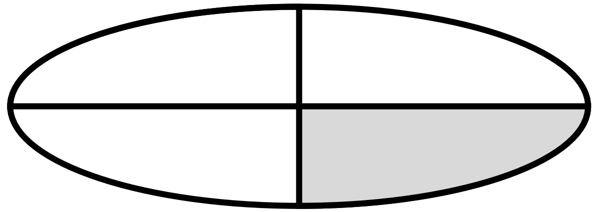
$$90 = \underbrace{2 \times 3 \times \rlap{\overbrace{}^{15}}3}_{18} \times 5$$

Week 5 Session 1: Part of a whole

We can use fractions to describe **equal parts** of a whole.



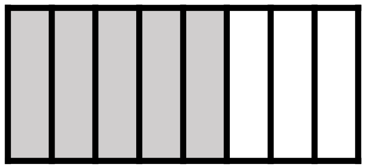
This is not $\frac{1}{4}$



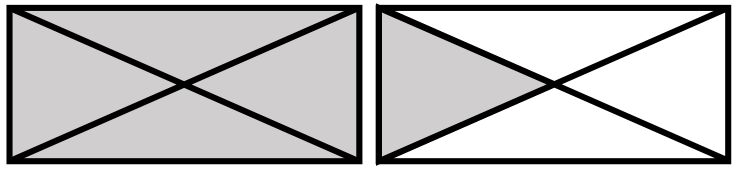
This is $\frac{1}{4}$

Complete the statements below:

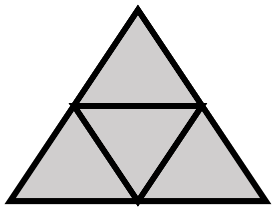
This is 1



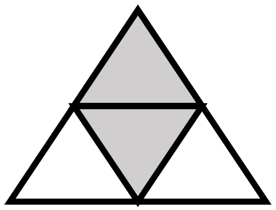
This is $\frac{?}{8}$



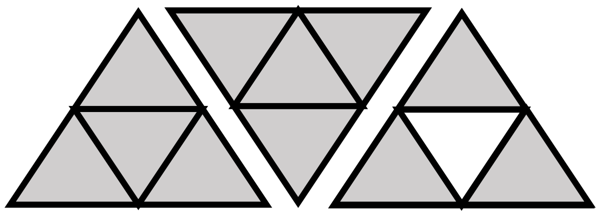
This is $\frac{5}{?} = 1\frac{1}{?}$



This is 1



This is…



This is…

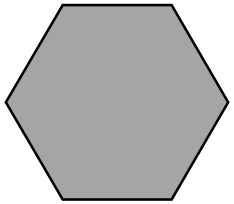
Week 5 Session 1: Part of a whole

Complete the statements for each set of shapes

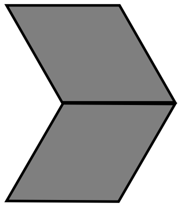
Example:

= one half

$= \frac{1}{2}$

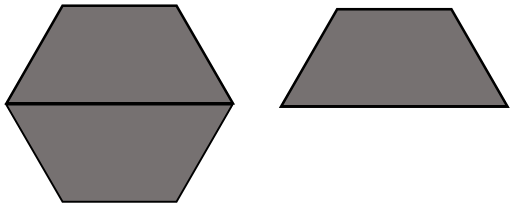


= one whole = 1 hexagon



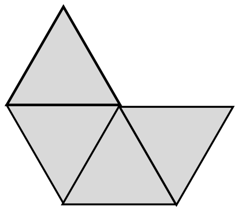
= two_______

= —



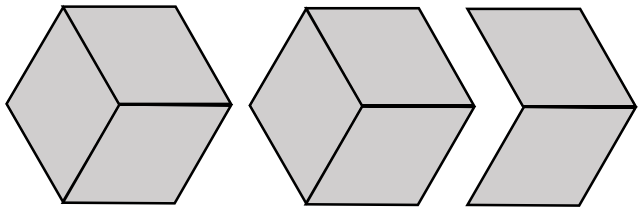
= ___________

=



= ___________

=



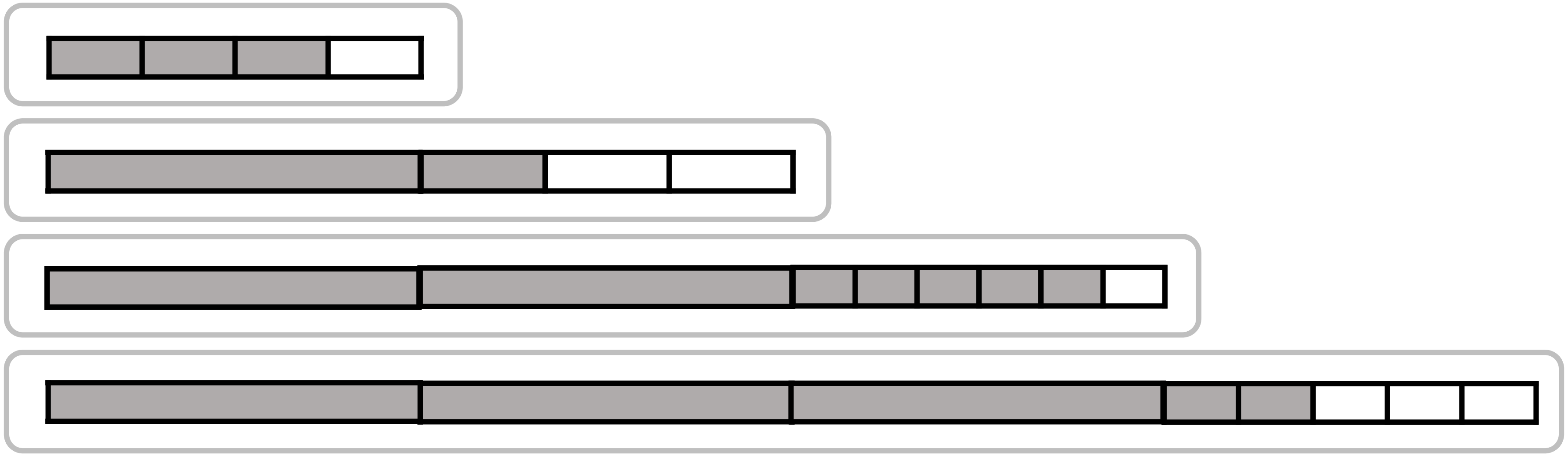
= __________

=

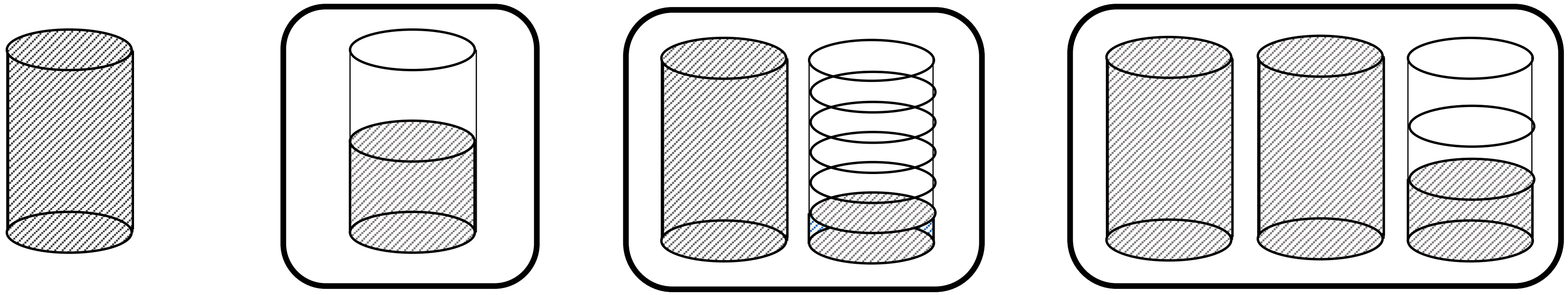
Week 5 Session 2: Fractions of measure

This is a fully-shaded 1 metre bar: 

What fraction of a 1 metre bar is shaded in each diagram?



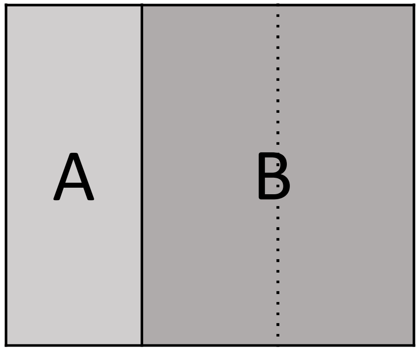
What fraction of the 1l of orange juice is shown in each diagram?

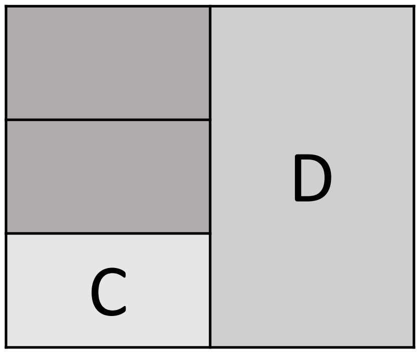


1l

This rectangle represents a farm of area 6 acres.

The diagrams below show the farm divided up in two different ways.





Use the diagrams to complete the statements:

Section A is ____ of the farm or **2** acres

Section B is ____ of the farm or ____ acres

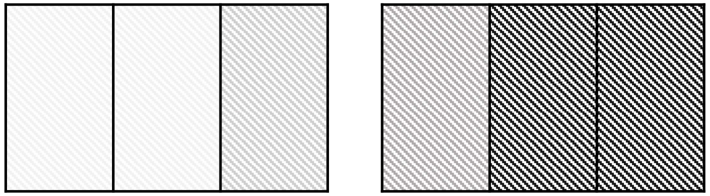
Section C is ____ of the farm or ____ acres

Section D is $\mathbf{\frac{1}{2}}$ of the farm or ____ acres

Week 5 Session 3: Fair shares

Two bars of chocolate are shared **equally** by three children.

They get $\frac{2}{3}$ of a bar each.



Use some scrap paper to see if you can share two chocolate bars between three children using different cuts.

What would happen to the amount of chocolate each child gets if…

a) The number of children they are sharing between goes up
b) The number of chocolate bars they have goes up

Week 5 Session 3: Fair shares

Look at how chocolate is shared in the two groups below.

Group A	Group B
Five bars of chocolate are shared equally by two children.	Seven bars of chocolate are shared equally by three children.

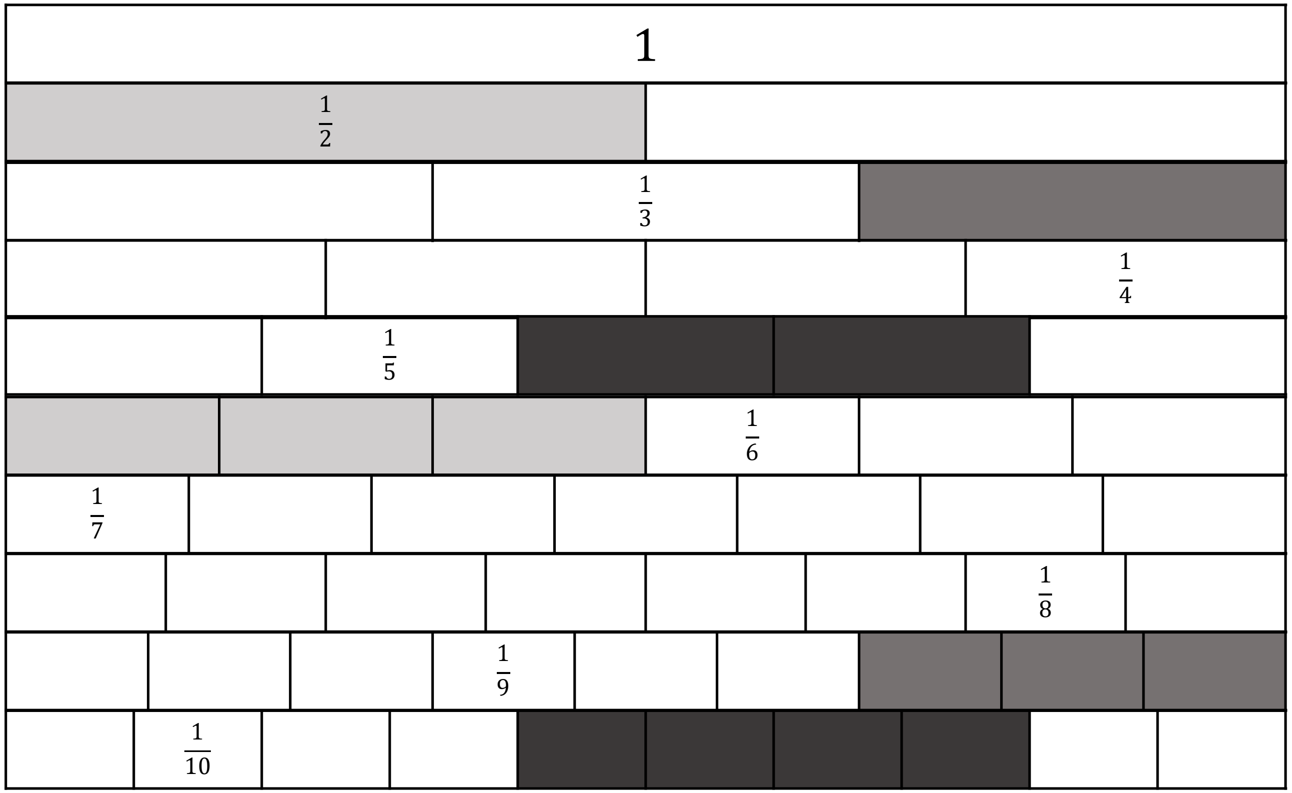
Who gets more chocolate? Use a diagram to help explain your answer.

Try different numbers of chocolate bars and different numbers of children.

Can you create two **different** groups where each child gets the **same** amount of chocolate?

Week 5 Session 4: Equivalence

There are many ways to write fractions that represent the same value



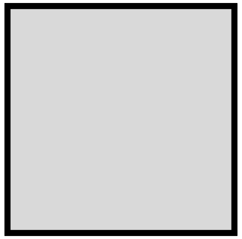
$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{\boxed{}}{9}$$

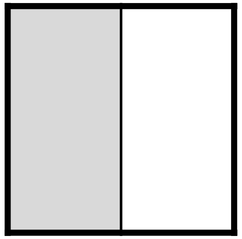
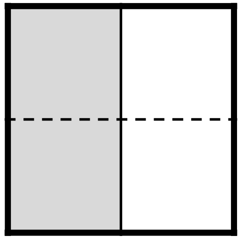
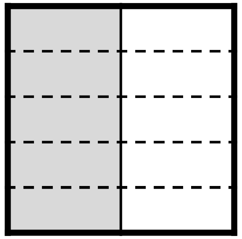
$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

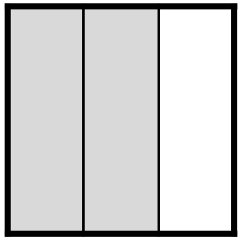
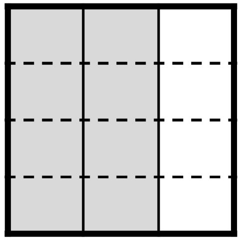
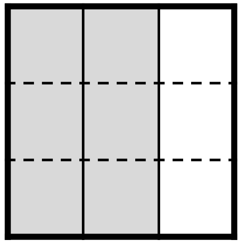
Find other equivalent fractions from the diagram

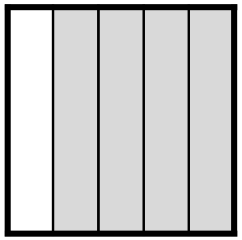
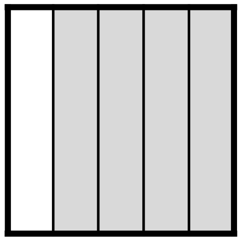
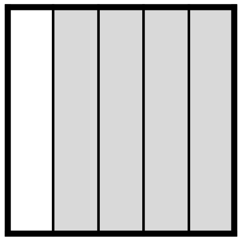
Week 5 Session 4: Equivalence

 = 1

Fill in the blanks below

 $\frac{1}{2}$ =  $\frac{2}{\square}$ =  $\frac{\square}{10}$

 $\frac{\square}{\square}$ =  $\frac{8}{12}$ =  $\frac{\square}{\square}$

 $\frac{4}{5}$ =  $\frac{8}{\square}$ =  $\frac{\square}{\square}$